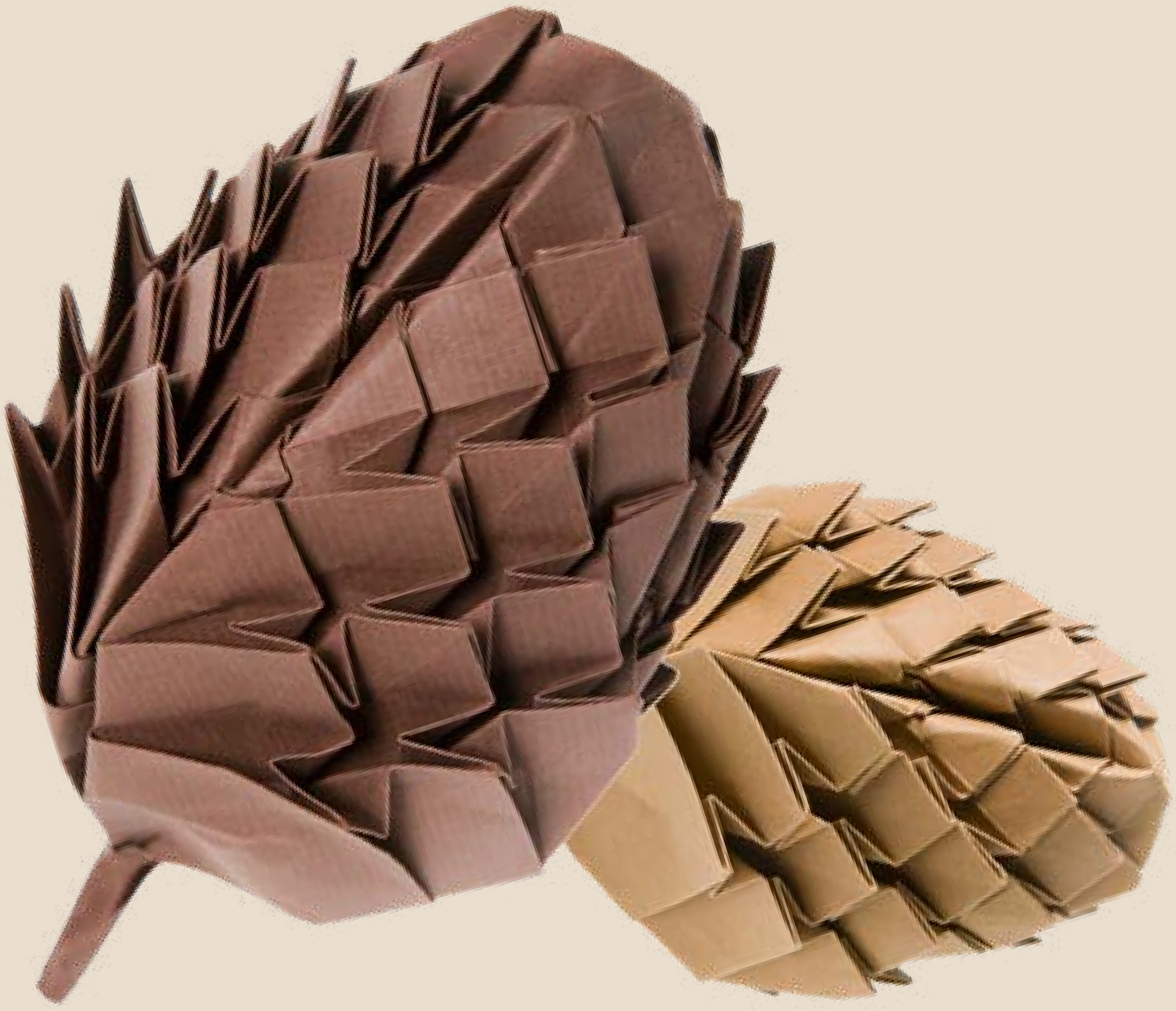
# AEA Mathematics



## Sample Assessment Materials

**Pearson Edexcel Advanced Extension Award in Mathematics**

*First teaching from September 2017*

*First certification from 2019* | Issue 1

**Edexcel, BTEC and LCCI qualifications**

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Original Origami Artwork designed by Beth Johnson and folded by Mark Bolitho
Origami photography: Pearson Education Ltd/Naki Kouyioumtzis

ISBN 978 1 4469 5992 3

# Contents

# Introduction

The Pearson Edexcel Advanced Extension Award in Mathematics is designed for use in schools and colleges. It is part of a suite of AS/A Level qualifications offered by Pearson Edexcel.

These sample assessment materials have been developed to support this qualification and will be used as the benchmark to develop the assessment students will take.

The booklet '*Mathematical formulae and statistical tables*' should be used for this assessment and can be downloaded from our website, qualifications.pearson.com.

# General marking guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than be penalised for omissions.
- Examiners should mark according to the mark scheme – not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive. However different examples of responses will be provided at standardisation.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed-out work should be marked **unless** the candidate has replaced it with an alternative response.

### Specific guidance for mathematics

1. These mark schemes use the following types of marks:
   - **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks: Unconditional accuracy marks, independent of M marks
   - **S** marks: For mathematical style and clarity in the presentation of solutions.
   - Marks should not be subdivided.
2. Abbreviations

   These are some of the traditional marking abbreviations that may appear in the mark schemes.

   - **bod** benefit of doubt
   - **ft** follow through
   - $\surd$ this symbol is used for correct ft
   - **cao** correct answer only
   - **cso** correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** ignore subsequent working
   - **awrt** answers which round to
   - **SC:** special case
   - **o.e.** or equivalent (and appropriate)
   - **d…** or **dep** dependent
   - **indep** independent
   - **dp** decimal places
   - **sf** significant figures

- ✱ The answer is printed on the paper or ag- answer given
- ⎾ or d… The second mark is dependent on gaining the first mark

3. All M marks are follow through.

   All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

   The style and clarity of presentation marks will be awarded as indicated in the mark scheme. These are denoted as S marks and will be awarded for clear, elegant and succinct complete solutions to questions. There will be clear indicators in the mark scheme as to where S marks will be awarded.

   An example of indicators of how S marks will be seen and awarded in the mark scheme is shown in the table below.

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **4(a)** | (Parallel to plane): $1.2W\cos\theta = 0.6W$ | Resolves parallel to plane with correct value of sin $\alpha$ used. **(S+)** | **M1** | AO3 |
| | $\theta = \frac{\pi}{3}$ | Accept 60° **(S−)** | **A1** | AO3 |
| **S1** | S1 mark: Award S1 for a clear and concise solution that is either:<br>- fully correct and does not include an S− point.<br>or<br>- that scores 11/12 and includes an S+ point but not an S− point. | | | |

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternative answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but deemed to be valid, examiners must escalate the response to a senior examiner to review.

Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel Award**

Centre Number

Candidate Number

**Sample Assessment Material for first teaching September 2017**

Time: **3 hours**

Paper Reference **9811/11**

# Advanced Extension Award Mathematics

**You must have:**
Mathematical Formulae and Statistical Tables
An insert for Question 6 and 7

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- **Calculators may not be used**.
- You must **show all your working**.
- Answers should be given in as simple a form as possible. e.g. $\frac{2\pi}{3}$, $\sqrt{2}$, $3\sqrt{2}$.

## Information

- The total mark for this paper is 100 of which **7** marks are for style and clarity of presentation.
- The style and clarity of presentation marks will be indicated as **(+S1) or (+S2)**.
- There are 7 questions in this question paper.
- The marks for each question are shown in brackets.
- The total mark for each question is shown at the end of the question.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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S 6 3 2 2 1 A 0 1 3 2

**Answer ALL questions. Write your answers in the spaces provided.**

**1.** (a) For $|y| < 1$, write down the binomial series expansion of $(1 - y)^{-2}$ in ascending powers of $y$ up to and including the term in $y^3$

**(1)**

(b) Show that when it is convergent, the series

$$1 + \frac{2x}{x+3} + \frac{3x^2}{(x+3)^2} + \ldots + \frac{rx^{r-1}}{(x+3)^{r-1}} + \ldots$$

can be written in the form $(1 + ax)^n$, where $a$ and $n$ are constants to be found.

**(4)**

(c) Find the set of values of $x$ for which the series in part (b) is convergent.

**(3)**

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**Question 1 continued**

**(Total for Question 1 is 8 marks)**

**2.** (a) On separate diagrams, sketch the curves with the following equations. On each sketch you should label the exact coordinates of the points where the curve meets the coordinate axes.

(i) $y = 8 + 2x - x^2$

(ii) $y = 8 + 2|x| - x^2$

(iii) $y = 8 + x + |x| - x^2$

**(7)**

(b) Find the values of $x$ for which

$$\left|8 + x + |x| - x^2\right| = 8 + 2|x| - x^2$$

**(4)**

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**Question 2 continued**

**Question 2 continued**

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**Question 2 continued**

**(Total for Question 2 is 11 marks)**

**3.**

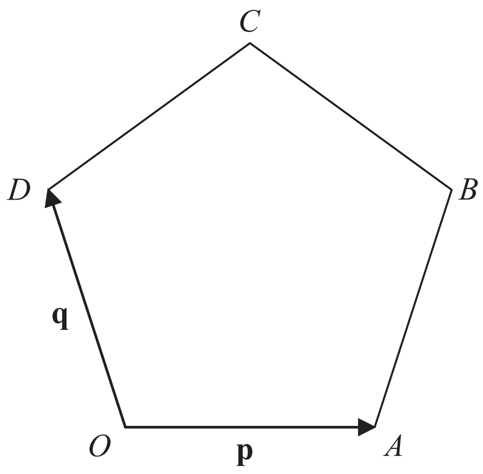


**Figure 1**

Figure 1 shows a regular pentagon $OABCD$. The vectors **p** and **q** are defined by $\mathbf{p} = \overrightarrow{OA}$ and $\mathbf{q} = \overrightarrow{OD}$ respectively.

Let $k$ be the number such that $\overrightarrow{DB} = k\overrightarrow{OA}$.

(a) Write down $\overrightarrow{AC}$ in terms of **p**, **q** and $k$ as appropriate.

**(1)**

(b) Show that $\overrightarrow{CD} = -\mathbf{p} - \dfrac{1}{k}\mathbf{q}$

**(3)**

(c) Hence find the value of $k$

**(4)**

**(+S1)**

By considering triangle $DBC$, or otherwise,

(d) find the exact value of $\sin 54°$

**(3)**

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**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 12 marks)**

**4.**

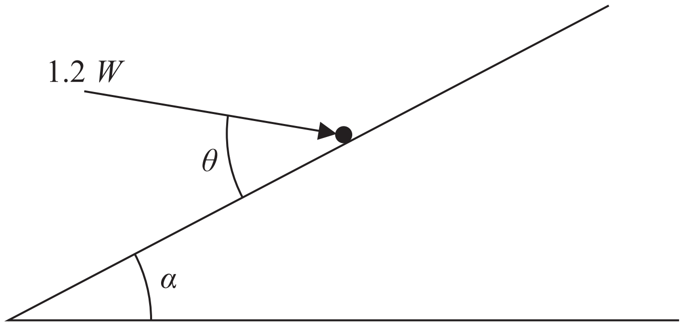


**Figure 2**

A particle of weight $W$ lies on a rough plane. The plane is inclined to the horizontal at an angle $\alpha$ where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the particle and the plane is $\frac{1}{2}$

The particle is held in equilibrium by a force of magnitude $1.2W$. The force makes an angle $\theta$ with the plane, where $0 < \theta < \pi$, and acts in a vertical plane containing a line of greatest slope of the plane, as shown in Figure 2.

(a) Find the value of $\theta$ for which there is no frictional force acting on the particle.

**(2)**

The minimum value of $\theta$ for the particle to remain in equilibrium is $\beta$

(b) Show that

$$\beta = \arccos\left(\frac{\sqrt{5}}{3}\right) - \arctan\left(\frac{1}{2}\right)$$

**(5)**

(c) Find the range of values of $\theta$ for which the particle remains in equilibrium with the frictional force acting up the plane.

**(5)**

**(+S1)**

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**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 13 marks)**

**5.**

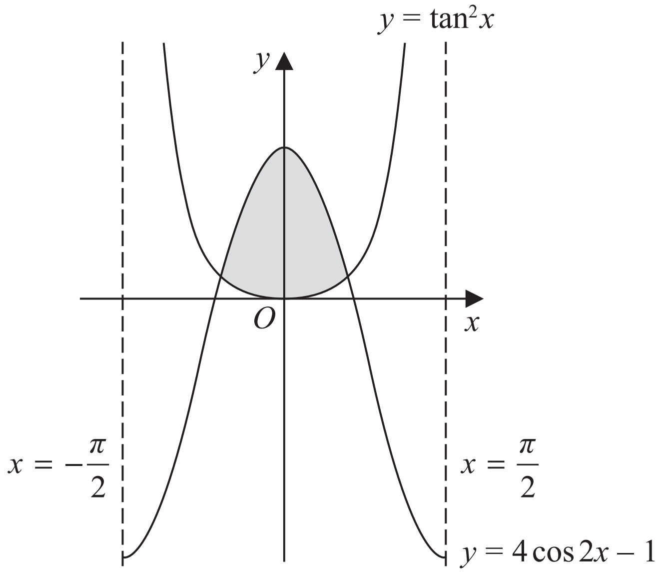


**Figure 3**

Show that the area of the finite region between the curves $y = \tan^2 x$ and $y = 4\cos 2x - 1$ in the interval $-\frac{\pi}{2} < x < \frac{\pi}{2}$, shown shaded in Figure 3, is given by

$$2\sqrt{2\sqrt{3}} - 2\sqrt{2\sqrt{3} - 3}$$

**(12)**

**(+S1)**

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**Question 5 continued**

**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 13 marks)**

**6.** (i) Eden, who is confused about the laws of logarithms, states that

$$(\log_5 p)^2 = \log_5(p^2)$$

and $\log_5 (q - p) = \log_5 q - \log_5 p$

However, there is a value of $p$ and a value of $q$ for which both statements are correct.

Determine these values.

**(7)**

(ii) (a) Let $r \in \mathbb{R}^+$, $r \neq 1$. Prove that

$$\log_r A = \log_{r^2} B \Rightarrow A^2 = B$$

**(2)**

(b) Solve

$$\log_4 (3x^3 + 26x^2 + 40x) = 2 + \log_2(x + 2)$$

**(7)**

**(+S2)**

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**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 18 marks)**

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**7.**

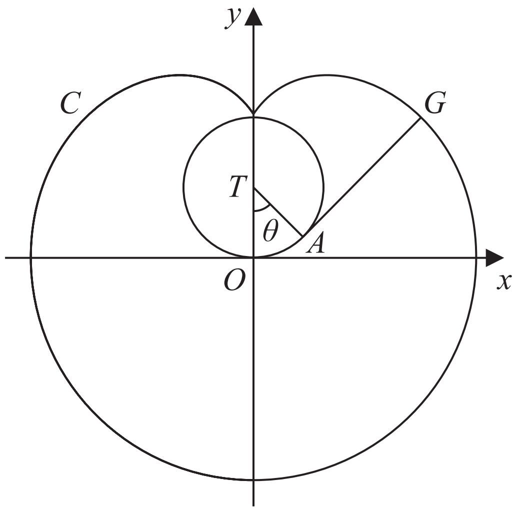


**Figure 4**

A circular tower of radius 1 metre stands in a large horizontal field of grass. A goat is attached to one end of a rope and the other end of the rope is attached to a fixed point $O$ at the base of the tower. The goat cannot enter the tower.

Taking the point $O$ as the origin (0, 0), the centre of the base of the tower is at the point $T(0, 1)$, where the unit of length is the metre.

The rope has length $\pi$ metres and you may ignore the size of the goat.

The curve $C$ shown in Figure 4 represents the edge of the region that the goat can reach.

(a) Write down the equation of $C$ for $y < 0$

**(1)**

When the goat is at the point $G(x, y)$, with $x > 0$ and $y > 0$, as shown in Figure 4, the rope lies along $OAG$ where $OA$ is an arc of the circle with angle $OTA = \theta$ radians and $AG$ is a tangent to the circle at $A$.

(b) With the aid of a suitable diagram show that

$$x = \sin\theta + (\pi - \theta)\cos\theta$$
$$y = 1 - \cos\theta + (\pi - \theta)\sin\theta$$

**(5)**

**Question 7 continued**

(c) By considering $\int y\frac{\mathrm{d}x}{\mathrm{d}\theta}\mathrm{d}\theta$, show that the area, in the first quadrant, between $C$, the positive $x$-axis and the positive $y$-axis can be expressed in the form

$$\int_0^{\pi} u\sin u\,\mathrm{d}u + \int_0^{\pi} u^2\sin^2 u\,\mathrm{d}u + \int_0^{\pi} u\sin u\cos u\,\mathrm{d}u$$

**(5)**

(d) Show that $\int_0^{\pi} u^2\sin^2 u\,\mathrm{d}u = \frac{\pi^3}{6} + \int_0^{\pi} u\sin u\cos u\,\mathrm{d}u$

**(4)**

(e) Hence find the area of grass that can be reached by the goat.

**(8)**

**(+S2)**

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**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 25 marks)**

**FOR STYLE AND CLARITY OF PRESENTATION: 7 MARKS**
**TOTAL FOR PAPER IS 100 MARKS**

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Pearson Edexcel Award

Sample Assessment Material for first teaching September 2017

Paper Reference **9811/11**

# Advanced Extension Award Mathematics

**Insert of questions 6 and 7**

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**6.** (i) Eden, who is confused about the laws of logarithms, states that

$$(\log_5 p)^2 = \log_5(p^2)$$

and $\log_5 (q - p) = \log_5 q - \log_5 p$

However, there is a value of $p$ and a value of $q$ for which both statements are correct.

Determine these values.

**(7)**

(ii) (a) Let $r \in \mathbb{R}^+$, $r \neq 1$. Prove that

$$\log_r A = \log_{r^2} B \Rightarrow A^2 = B$$

**(2)**

(b) Solve

$$\log_4 (3x^3 + 26x^2 + 40x) = 2 + \log_2(x + 2)$$

**(7)**

**(+S2)**

**(Total for Question 6 is 18 marks)**

**7.**

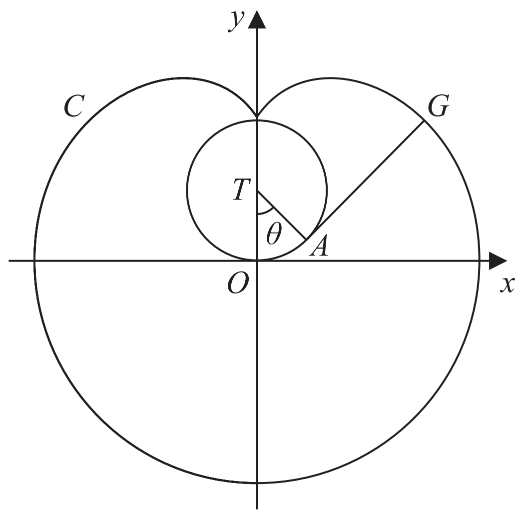


**Figure 4**

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The curve $C$ shown in Figure 4 represents the edge of the region that the goat can reach.

(a) Write down the equation of $C$ for $y < 0$

**(1)**

When the goat is at the point $G(x, y)$, with $x > 0$ and $y > 0$, as shown in Figure 4, the rope lies along $OAG$ where $OA$ is an arc of the circle with angle $OAG = \theta$ radians and $AG$ is a tangent to the circle at $A$.

(b) With the aid of a suitable diagram show that

$$x = \sin\theta + (\pi - \theta)\cos\theta$$
$$y = 1 - \cos\theta + (\pi - \theta)\sin\theta$$

**(5)**

**Question 7 continued**

(c) By considering $\int y \frac{\mathrm{d}x}{\mathrm{d}\theta} \mathrm{d}\theta$, show that the area, in the first quadrant, between $C$, the positive $x$-axis and the positive $y$-axis can be expressed in the form

$$\int_0^{\pi} u \sin u \, \mathrm{d}u + \int_0^{\pi} u^2 \sin^2 u \, \mathrm{d}u + \int_0^{\pi} u \sin u \cos u \, \mathrm{d}u$$

**(5)**

(d) Show that $\int_0^{\pi} u^2 \sin^2 u \, \mathrm{d}u = \frac{\pi^3}{6} + \int_0^{\pi} u \sin u \cos u \, \mathrm{d}u$

**(4)**

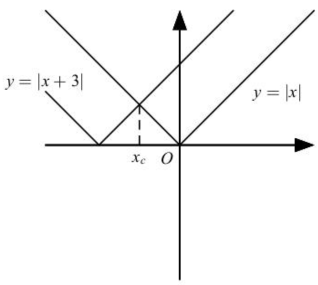
(e) Hence find the area of grass that can be reached by the goat.

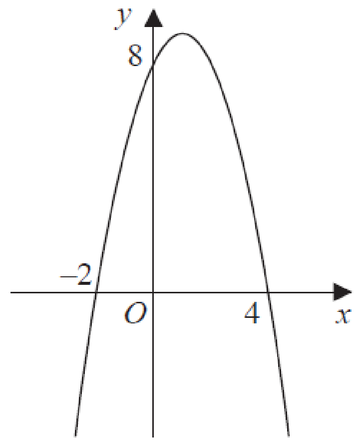
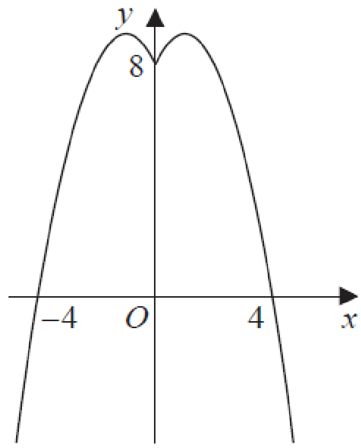
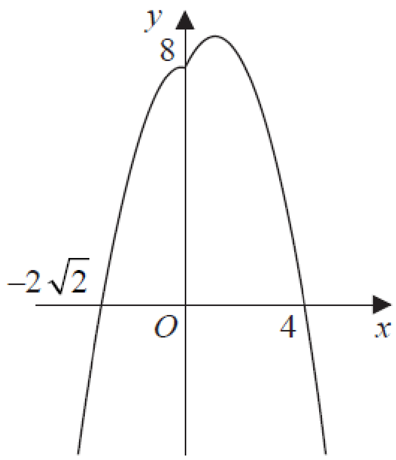
**(8)**

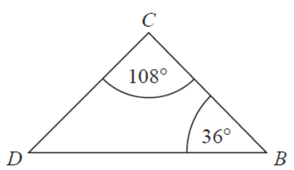
**(+S2)**

**(Total for Question 7 is 25 marks)**

## AEA Mark scheme

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **1(a)** | $(1-y)^{-2} = 1 + 2y + 3y^2 + 4y^3 + \ldots$ | | **B1** | AO1 |
| | | | **(1)** | |
| **(b)** | $S = 1 + 2\left(\frac{x}{x+3}\right) + 3\left(\frac{x}{x+3}\right)^2 + \cdots$ | Identifies $y = \frac{x}{x+3}$ | **M1** | AO1 |
| | $\Rightarrow S = 1 + 2y + 3y^2 + \cdots = \left(1 - \frac{x}{x+3}\right)^{-2}$ | Uses (a) to form correct expression. | **A1** | AO2 |
| | $= \left(\frac{x+3-x}{x+3}\right)^{-2} = \left(\frac{x+3}{3}\right)^2$ | Combines terms and reciprocates. | **M1** | AO2 |
| | $= \left(1 + \frac{1}{3}x\right)^2$ (so $a = \frac{1}{3}$ and $n = 2$) | Correct answer. | **A1** | AO2 |
| | | | **(4)** | |
| **(c)** | Need $\left\|\frac{x}{x+3}\right\| < 1$ or $\|x\| < \|x+3\|$ | Correct condition. | **B1** | AO1 |
| |  Critical value is when $x + 3 = -x$, so $x = \ldots$ | Any suitable strategy for critical value. | **M1** | AO3 |
| | So convergent on $\left\{x \in \mathbb{R} : x > -\frac{3}{2}\right\}$ Accept equivalent set notation forms, e.g. $\left\{x : x \in \mathbb{R}, x > -\frac{3}{2}\right\}$ or $\left\{x : x > -\frac{3}{2}\right\}$ or $\left(-\frac{3}{2}, \infty\right]$ | | **A1** | AO2 |
| | | | **(3)** | |
| | | | **(8 marks)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** | **(i)**  **(ii)**  | | |
| **(i)** | Correct shape and (0, 8) | **B1** | AO1 |
| | Crossing $x$-axis at −2 and 4 | **B1** | AO1 |
| **(ii)** | Symmetrical shape with 2 maxima and crosses at $\pm 4$ | **B1** | AO1 |
| | Correct shape at (0, 8) - not curved as a minimum | **B1** | AO3 |
| **(iii)** |  Correct for $x > 0$ and (4, 0) marked | **B1** | AO1 |
| | Correct for $x \leqslant 0$ with $-2\sqrt{2}$ marked and zero gradient at (0, 8) | **B1** | AO3 |
| | Clear "kink" at (0, 8) | **B1** | AO3 |
| | | **(7)** | |
| **(b)** | Considers $x \geqslant 0 : 0 \leqslant x \leqslant 4$<br>Correct inequality with both end points included. | **B1** | AO3 |
| | Considers $x < 0$ : attempts $-(8 - x^2) = 8 - 2x - x^2 \Rightarrow x = \ldots$ | **M1** | AO3 |
| | $x < 0,\ 2x^2 + 2x - 16 = 0 \Rightarrow x = \dfrac{-1-\sqrt{33}}{2}$ only (withhold A if second root given). | **M1**<br>**A1** | AO3<br>AO3 |
| | | **(4)** | |
| | | **(11 marks)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 3(a) | $\overrightarrow{AC} = k\mathbf{q}$ | **B1** | AO1 |
| | | **(1)** | |
| (b) | $\overrightarrow{BO} = \overrightarrow{BD} + \overrightarrow{DO} = -k\mathbf{p} - \mathbf{q}$ or other suitable vector equation leading to $\overrightarrow{BO}$ used. | **M1** | AO1 |
| | $\overrightarrow{BO} = k\overrightarrow{CD}$ or $\overrightarrow{CD} = \frac{1}{k}\overrightarrow{BO}$ or equivalent used (or implied by use). | **B1** | AO3 |
| | $\overrightarrow{CD} = \frac{1}{k}(-k\mathbf{p} - \mathbf{q}) = -\mathbf{p} - \frac{1}{k}\mathbf{q}$ * | **A1*** | AO2 |
| | | **(3)** | |
| (c) | Use of $\mathbf{p} + \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \mathbf{q}$ or equivalent vector equation used. | **B1** | AO3 |
| | $\Rightarrow \mathbf{p} + \overrightarrow{AC} + \overrightarrow{CD} = \mathbf{q}$<br>$\Rightarrow \mathbf{p} + k\mathbf{q} - \mathbf{p} - \frac{1}{k}\mathbf{q} = \mathbf{q} \Rightarrow (k^2 - k - 1)\mathbf{q} = \mathbf{0}$ | **M1** | AO3 |
| | $\Rightarrow k^2 - k - 1 = 0 \Rightarrow k = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$ | **M1** | AO3 |
| | (So as $k > 0$, we must have)<br>$k = \dfrac{1+\sqrt{5}}{2}$<br>**(S+)** | **A1** | AO2 |
| | | **(4)** | |
| (d) | <br>Use the symmetry of the shape to deduce relevant angle<br>e.g. $\angle BCD = 180° - 72° = 108°$. (May use another triangle within the shape.) | **B1** | AO1 |
| | Hence $\sin 54° = \dfrac{\frac{1}{2}DB}{BC} = \dfrac{\frac{1}{2}kOA}{OA} = \dfrac{1}{2}k$<br>Correct work in their triangle to reduce to an expression in $k$ | **M1** | AO3 |
| | $= \dfrac{1+\sqrt{5}}{4}$ | **A1** | AO3 |
| | | **(3)** | |
| S1 | S1 mark: Award S1 for a clear and concise solution that is either<br>- 11 marks with correct notation for vectors in the main.<br>or<br>- that scores 10+ and includes the S+ point but may have some poor notation. | **(1)** | AO2 |
| | | **(11 + 1 marks)** | |

**Notes:**

**(c)** S+ for clear consideration of which root is needed.

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **4(a)** | (Parallel to plane): $1.2W\cos\theta = 0.6W$ | Resolves parallel to plane with correct value of $\sin\alpha$ used. **(S+)** | **M1** | AO3 |
| | $\theta = \frac{\pi}{3}$ | Accept 60° **(S−)** | **A1** | AO3 |
| | | | **(2)** | |
| **(b)** | (Parallel to plane): $F = 1.2W\cos\theta - 0.6W$ | M resolves in both directions.<br>A for both correct. **(S+)** | **M1** | AO3 |
| | (Perpendicular to plane):<br>$R = 0.8W + 1.2W\sin\theta$ | | **A1** | AO3 |
| | Minimum value of $\theta$ when $F = \mu R$<br>$\Rightarrow 1.2\cos\theta - 0.6 = 0.4 + 0.6\sin\theta$ | Uses $F = \mu R$ to form equation in $\theta$ only. | **M1** | AO3 |
| | $\Rightarrow 2\cos\theta - \sin\theta = \frac{5}{3} \Rightarrow \sqrt{5}\cos(\theta+\gamma) = \frac{5}{3}$ | Applies $R\cos(\theta+\alpha)$ | **M1** | AO2 |
| | $\Rightarrow \theta = \arccos\left(\frac{\sqrt{5}}{3}\right) - \arctan\left(\frac{1}{2}\right)$ * | | **A1*** | AO2 |
| | | | **(5)** | |
| **(c)** | (Parallel to plane): $F = 0.6W - 1.2W\cos\theta$<br>(Perpendicular to plane): $R = 0.8W + 1.2W\sin\theta$ | Resolves both directions. | **M1** | AO3 |
| | $F = \mu R \Rightarrow 0.6W - 1.2W\cos\theta = 0.4W + 0.6W\sin\theta$ | Correct equation using $F = \mu R$ | **A1** | AO3 |
| | $\Rightarrow 2\cos\theta + \sin\theta = \frac{1}{3} \Rightarrow \sqrt{5}\cos(\theta-\gamma) = \frac{1}{3}$<br>OR $\frac{6t}{1+t^2} + \frac{6(1-t^2)}{1+t^2} = 1 \Rightarrow 7t^2 - 6t - 5 = 0$ | Correct strategy for finding $\theta$ | **M1** | AO3 |
| | Greatest value of<br>$\theta = \arccos\left(\frac{\sqrt{5}}{15}\right) + \arctan\left(\frac{1}{2}\right)$<br>OR $\theta = 2\arctan\left(\frac{3+2\sqrt{11}}{7}\right)$ | Either form for $\theta$ | **A1** | AO3 |

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **4(c)** *continued* | So range is $\frac{\pi}{3} < \theta \leqslant \arccos\left(\frac{\sqrt{5}}{15}\right) + \arctan\left(\frac{1}{2}\right)$ (oe) | Allow `least value' of $\theta$ is $\frac{\pi}{3}$ ($\frac{\pi}{3} \leqslant \theta$) i.e lower strict inequality not required for this mark but incorrect inequality is. **(S−)** | **A1** | AO2 |
| | | | **(5)** | |
| **S1** | S1 mark: Award S1 for a clear and concise solution that is either<br>- fully correct and does not include an S− point.<br>or<br>- that scores 11/12 and includes an S+ point but not an S− point. | | **(1)** | AO2 |
| | | | **(12 + 1 marks)** | |

**Notes:**

| | |
|---|---|
| **(a)+(b)** | S+ for good reasoning for the angle, good use of diagrams etc.<br>S− for giving answer in degrees. |

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| 5 | $\tan^2 x = 4\cos 2x - 1$<br>$\Rightarrow \sec^2 x - 1 = 4(2\cos^2 x - 1) - 1$<br>OR $\dfrac{\sin^2 x}{\cos^2 x} = 4(2\cos^2 x - 1) - 1$ | Reduces RHS to single angle and adapts $\tan^2 x$ suitably. | M1 | AO1 |
| | $\Rightarrow\ 1 - \cos^2 x = 8\cos^4 x - 5\cos^2 x$ | Achieve equation in just $\cos x$ | M1 | AO1 |
| | $\Rightarrow 8\cos^4 x - 4\cos^2 x - 1 = 0$ | Correct quartic. | A1 | AO1 |
| | $\Rightarrow \cos^2 x = \dfrac{4 \pm \sqrt{16 - 4\times 8 \times -1}}{16} = \ldots$ | Attempts roots. | M1 | AO3 |
| | Intersection point $\alpha$ satisfies<br>$\cos^2\alpha = \dfrac{1}{4}\left(1+\sqrt{3}\right)$ | Correct root identified. **(S+)** | A1 | AO3 |
| | So $\sin^2\alpha = 1 - \dfrac{1}{4}\left(1+\sqrt{3}\right) = \dfrac{1}{4}\left(3-\sqrt{3}\right)$ | Correct expressions for $\sin^2\alpha$ or $\sin\alpha$ and $\tan^2\alpha$ or $\tan\alpha$ seen anywhere in solution. | B1 | AO2 |
| | And $\tan^2\alpha = \dfrac{3-\sqrt{3}}{1+\sqrt{3}} = 2\sqrt{3} - 3$ | | B1 | AO2 |
| | Area = $2\times\displaystyle\int_0^\alpha \left(4\cos 2x - 1 - \tan^2 x\right) \mathrm{d}x$ | Correct strategy for the area, use of 0 and $\alpha$ with 2×, or with $-\alpha$ and $\alpha$. May integrate separately and combine later. **(S+)(S−)** | M1 | AO3 |
| | $= 2\times\displaystyle\int_0^\alpha \left(4\cos 2x - 1 - (\sec^2 x - 1)\right) \mathrm{d}x$ | Uses suitable identity to set up integral (ignore 2× and limits here). | M1 | AO3 |
| | $= 2\times\left[2\sin(2x) - \tan x\right]_0^\alpha$ | Correct integral (ignore limits etc.) | A1 | AO3 |
| | $= 2\times\left(4\sin\alpha\cos\alpha - \tan\alpha\right)$<br>$= 2\times\left(4\times\dfrac{1}{2}\sqrt{3-\sqrt{3}}\times\dfrac{1}{2}\sqrt{1+\sqrt{3}} - \sqrt{2\sqrt{3}-3}\right)$ | Uses the expression in $\cos\alpha$, $\sin\alpha$ and $\tan\alpha$ in their integral. | M1 | AO3 |

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **5** *continued* | $= 2\sqrt{2\sqrt{3}} - 2\sqrt{2\sqrt{3} - 3}$ * | Completes to given answer. | **A1*** | AO2 |
| | | | **(12)** | |
| **S1** | S1 mark: Award S1 for a clear and concise solution that is either<br>- fully correct with no S− point.<br>or<br>- that scores 11/12 and includes an S+ point and no S−. | | **(1)** | AO2 |
| | | | **(12+1 marks)** | |

**Notes:**

**S+** for showing consideration and rejection of the negative root for $\cos^2 x$

**S+** for good use of symmetry and/or odd and even function nature of the trig functions.

**S−** if the strategy is cumbersome – e.g. Separate integrals, no use of symmetry etc.

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **6(i)** | $(\log_5 p)^2 = \log_5(p^2) \Rightarrow (\log_5 p)^2 = 2\log_5 p$ | Applies power rule. | **M1** | AO1 |
| | $\Rightarrow \log_5 p(\log_5 p - 2) = 0 \Rightarrow \log_5 p = 0 \therefore p = 1$ | | **A1** | AO2 |
| | or $\Rightarrow \log_5 p = 2 \ \therefore\ p = 25$ | | **A1** | AO2 |
| | $\log_5(q-p) = \log_5 q - \log_5 p$<br>$\Rightarrow \log_5(q-p) = \log_5\left(\frac{q}{p}\right)$ | Applies correct rule. | **M1** | AO1 |
| | $\Rightarrow q - p = \frac{q}{p}$ | | **A1** | AO2 |
| | $(p = 1 \Rightarrow q - 1 = q$ is impossible$)$ so $q = \frac{p^2}{p-1}$ | Makes $q$ the subject, no need to see $p = 1$ rejected for M<br>**(S+)** | **M1** | AO3 |
| | So $p = 25$ and $q = \frac{625}{24}$ (only) | | **A1** | AO3 |
| | | | **(7)** | |
| **(ii)(a)** | $\log_r(A) = \log_{r^2}(B) \Rightarrow (r^2)^{\log_r(A)} = (r^2)^{\log_{r^2}(B)}$ | Adopts a correct strategy, raise both sides to base $r^2$ or use change of base. | **M1** | AO2 |
| | $\Rightarrow r^{2\log_r(A)} = B \Rightarrow r^{\log_r(A^2)} = B \Rightarrow A^2 = B$ * | Achieves result correctly. | **A1*** | AO2 |
| | | | **(2)** | |
| **(ii)(b)** | $2 = \log_2(4)$ or $2 = \log_4(16)$ | Seen anywhere. | **B1** | AO1 |
| | $\therefore \log_4(3x^3 + 26x^2 + 40x) = \log_2(4(x+2))$ (oe) | Combines to just two log terms. | **M1** | AO3 |
| | $\therefore 3x^3 + 26x^2 + 40x = 16(x+2)^2$ | Applies the result of (ii)(a) | **M1** | AO3 |
| | e.g. $x(3x+20)(x+2) = 16(x+2)^2$<br>or $3x^3 + 10x^2 - 24x - 64 = 0$ leading to<br>$(x+2)(3x^2 + kx \pm 32) = 0$ | Identifying a linear factor. | **M1** | AO3 |
| | $(x+2)(\underline{3x^2 + 4x - 32}) = 0$ | Correct quadratic reached (no need for $(x + 2)$ for A1<br>**(S+)** | **A1** | AO3 |

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **6(ii)(b)** *continued* | $(x + 2)(3x - 8)(x + 4) = 0$ | Factorises | **M1** | AO3 |
| | But equation not defined for ($x = -2$ and) $x = -4$ <br> $\left(\text{but equation works for } x = \frac{8}{3}\right)$. So $x = \frac{8}{3}$ is only solution. <br> **(S+)** | | **A1** | AO3 |
| | | | **(7)** | |
| **S2** | Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below).<br>Award S1 for either<br>- a fully correct solution that is succinct but does not mention any S+ points.<br>or<br>- a fully correct solution that is slightly laboured but includes an S+ point.<br>or<br>- part (i) or (ii) fully correct and contains an S+ point. | | **(2)** | AO2 |
| | | | | **(16 + 2 marks)** |

**Notes:**

**(i)** **S+** for giving reason $p \neq 1$

**(ii)** **S+** for dealing correctly with $(x + 2)$ and not just "cancelling" – if statement $x \neq -2$ is given, cancelling is fine.

**S+** for correct reasoning for the only valid root with check that equation does work for the given value.

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| 7(a) | $x^2 + y^2 = \pi^2$ or equivalent. | | B1 | AO1 |
| | | | (1) | |
| (b) | $OA = \theta$ seen or implied. | | B1 | AO1 |
| | $AG = \pi - \theta$ seen or implied. | | B1 | AO1 |
| | Clear method for $x$ or $y$ e.g.<br>Let $X$ be the point vertically below $G$ such that angle $GXA$= 90°<br>OR identifies $x = \sin\theta + GA\cos\theta$ or $y = GA\sin\theta + 1 - \cos\theta$<br>**(S+)** | | M1 | AO2 |
| | So $x = TA\sin\theta + AX = \sin\theta + (\pi - \theta)\cos\theta$ * | | A1* | AO2 |
| | and $y = 1 - TA\cos\theta + GX = 1 - \cos\theta + (\pi - \theta)\sin\theta$ * | | A1* | AO2 |
| | | | (5) | |
| (c) | $\frac{dx}{d\theta} = \cos\theta - (\pi - \theta)\sin\theta - \cos\theta$ (allow one slip) | | M1 | AO1 |
| | So Area $\int_{x=0}^{x=\pi} y\frac{dx}{d\theta}\,d\theta = \int_{\pi}^{0}\left[1 - \cos\theta + (\pi - \theta)\sin\theta\right]\left[-(\pi - \theta)\sin\theta\right]\,d\theta$<br>(ignore limits) | | A1 | AO1 |
| | Identifies suitable substitution: Let $u = \pi - \theta$ | | M1 | AO3 |
| | Applies $\cos(\pi - \theta) = -\cos\theta$ and $\sin(\pi - \theta) = \sin\theta$ to their integral. | | M1 | AO3 |
| | Simplifies to given answer, with limits correctly derived.<br>Area $= -\int_0^{\pi}\left(-u\sin u - u\cos u\sin u - u^2\sin^2 u\right)\,du \to$ ans*<br>**(S+)** | | A1* | AO2 |
| | | | (5) | |
| (d) | $\int_0^{\pi} u^2\sin^2 u\,du = \int_0^{\pi}\frac{u^2}{2}\,du - \int_0^{\pi}\frac{u^2}{2}\cos 2u\,du$ | Use of $\sin^2 u$ in terms of $\cos 2u$ | M1 | AO2 |
| | $= \frac{\pi^3}{6}, -\left\{\left[\frac{u^2}{4}\sin 2u\right]_0^{\pi} - \int_0^{\pi} u\frac{\sin 2u}{2}\,du\right\}$ | achieves $\frac{\pi^3}{6}$ | A1cso | AO2 |
| | | Use integration by parts on second integral. | M1 | AO2 |
| | $= \frac{\pi^3}{6} + \int_0^{\pi} u\sin u\cos u\,du$ * | Shows [...] = 0 and simplifies to answer. | A1* | AO2 |
| | | | (4) | |

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **7(d)** *continued* | **Alternative** | | | |
| | $\int \sin^2 u \, du = \int \frac{1}{2} - \frac{1}{2}\cos 2u \, du = \frac{1}{2}u \pm \frac{1}{4}\sin 2u$ | Use of $\sin^2 u$ in terms of cos $2u$ and integrates to use in parts. | **M1** | AO2 |
| | $\int_0^\pi u^2 \sin^2 u \, du = \left[u^2\left(\frac{1}{2}u - \frac{1}{4}\sin 2u\right)\right]_0^\pi - \int_0^\pi 2u\left(\frac{1}{2}u - \frac{1}{4}\sin 2u\right) du$ | | | |
| | $= \frac{\pi^3}{2} - \int_0^\pi u^2 \, du + \int_0^\pi \frac{1}{2}\sin 2u \, du$ | achieves $\frac{\pi^3}{6}$ | **A1cso** | AO2 |
| | $= \frac{\pi^3}{6}$ | Having attempted parts directly. | **M1** | AO2 |
| | $= \frac{\pi^3}{6} + \int_0^\pi u \sin u \cos u \, du$ * | Correct work and simplifies to answer. | **A1*** | AO2 |
| | | | **(4)** | |
| **(e)** | $\int_0^\pi u \sin u \, du = [-u\cos u]_0^\pi + \int_0^\pi \cos u \, du$ | Use of parts on this integral (ignore limits). | **M1** | AO2 |
| | $= \pi$ | | **A1** | AO1 |
| | $\int_0^\pi u \sin u \cos u \, du = \int_0^\pi u \frac{\sin 2u}{2} du = \left[-u\frac{\cos 2u}{4}\right]_0^\pi + \int_0^\pi \frac{\cos 2u}{4} du$ | | **M1** | AO2 |
| | $= -\frac{\pi}{4}$ | M1 – use of parts<br>A1 – correct result | **A1** | AO1 |
| | Area between curve and +ve axes $= \frac{\pi^3}{6} + \pi - 2\times\frac{\pi}{4} = \underline{\frac{\pi^3}{6} + \frac{\pi}{2}}$ | | **A1** | AO2 |
| | Total area available = – tower + semicircle | Suitable strategy. **(S+)** | **M1** | AO3 |
| | Area of semicircle $= \frac{\pi^3}{2}$ <u>or</u> area of tower's base = $\pi$ | | **B1** | AO2 |
| | So area reachable is $\underline{\frac{5\pi^3}{6}}$ <u>m²</u> | | **A1** | AO3 |
| | | | **(8)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7(e)** *continued*<br><br>**S2** | Award S2 for a fully correct solution that is succinct and includes some S+ points (see notes below).<br>Award S1 for either<br>- a fully correct solution that is succinct but does not mention any S+ points.<br>or<br>- a fully correct solution that is slightly laboured but includes an S+ point.<br>or<br>- a score of >20 but solution is otherwise succinct or contains an S+ point. | **(2)** | AO2 |
| | | | **(23 + 2 marks)** |

**Notes:**

**(b)** **S+** for a clear and well expressed explanation or clear diagram with accompanying work.
**(c)** **S+** for clear demonstration of the change of limits.
**(e)** **S+** for a concise strategy using symmetry.

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ISBN 978-1-4469-3431-9